Newton's Third law; Action and Reaction are not always equal and Second law ; Mass m $=\frac{F}{a}=\frac{0}{0}$

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Abstract

Newton's laws have been quoted from the Philosophiæ Naturalis Principia Mathematica published in 1687. It is interesting to note that Newton has given second law of motion in different way and currently used by scientists in somewhat different way. An example has been quoted illustrating that action and reaction are not equal and opposite. This demonstration is repeatable. Newton's second law of motion reduces to first and third law if a =0 and F=0 (motion is un-accelerated). Thus it is called real law of motion. But under this condition the inertial mass is m $=\frac{F}{a}=\frac{0}{0}$, which is not justified. The inertial mass will be non-zero if a>0, F>0 or motion is accelerated, which changes basic notion of motion.

1.0 Newton's Principia and laws of motion

Newton's most famous book popularly known as **the Principia** was written originally in Latin as Philosophiæ Naturalis Principia Mathematica in 1687. It was translated to English by Andrew Motte in 1729 as "Mathematical Principles of Natural Philosophy". Two full English translations of Newton's 'Principia' have appeared, both based on Newton's 3rd edition of 1726. Newton three laws of motion in original Latin and English are given below.

Newton's third law of motion as stated in original Latin

Lex III: *Actioni contrariam semper et æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales et in partes contrarias dirigi.*

When translated to English

Law III: To every action there is always an equal and opposite reaction: or the forces of two bodies on each other are always equal and are directed in opposite directions.

Newton's second law of motion as stated in original Latin

Lex II: *Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur*.

Translated to original English

Law II: The alteration of motion is ever proportional to the motive force impress'd; and is made in the direction of the right line in which that force is impress'd.

2.0 Action and reaction are not always equal and opposite

Consider a body A of weight W is lying over the surface of the other body B . The body A exerts force W on the other body B. Thus,

Action of body A = W

According to Newton's third law of motion the body B , exerts equal and opposite reaction R to body A

Reaction of body B =R

Now both acting and reacting body remain at respective positions, thus

Action of body A = -Reaction of body B

or $R = -W$ (1)

If in general body A exerts force F_{AB} on body B , then body B exerts force F_{BA} on body A then according to Newton's third law of motion

$F_{AB} = -F_{BA}$ (2)

Thus action and reaction are equal and opposite.

2.1 Following example is neglected

Consider a boy is standing in front of concrete wall, holding a rubber ball and cloth ball in the hands. Let the wall is at the distance of 10m from the boy.

First case: Let the boy throws the rubber ball at the wall with force F. The ball strikes the wall, and comes back to the boy i.e. travelling 10m. Now action and reaction can be understood as

Action: Boy pushes the ball towards wall

Reaction: The wall pushes the ball towards boy.

As the boy pushed the ball from distance of 10m, and wall rebounds to distance of 10m. So action and reaction are equal and opposite.

Boy pushes the ball through 10m = Wall rebounds the ball through 10m

Action = -Reaction (3)

The –ve sign indicates that after reaction ball moves in opposite direction. In this case Newton's third law of motion completely obeyed.

Second case: Let the boy throws the cloth ball at the wall which is at distance of 10m.

Let the boy throws the cloth wall with same force F. The ball strikes the wall, and rebounds to only 5m. Now action and reaction can be understood as

Action: Boy pushes the ball towards wall

Reaction: The wall pushes the ball towards boy

As the boy pushed the cloth ball from distance of 10m, and wall returns to distance of 5m.

Thus boy pushes the ball towards the wall through 10m

The wall rebounds the ball through 5m

Action $\neq$ Reaction

So action and reaction are opposite but not equal in case of cloth ball. Thus reaction also depends upon various characteristics body or ball (cloth, rubber) and wall (concrete or wooden or earthen) . Thus reaction depends upon various characteristics of the system. It is just possible that characterises of ball and wall are such that ball may rebound more than 10m.

Generalization: It allows us to reassess Newton's third law of motion as[2]

" To every action there is always a opposite reaction which may or may be equal depending upon characteristics of the system."

3.0 Intricacies and inconsistencies results of second law of motion

Law II: The alteration of motion is ever proportional to the motive force impress'd; and is made in the direction of the right line in which that force is impress'd.

Here, the literary meaning of word alternation is

'Successive change from one thing or state to another and back again.'

In Alternating Current Generator emf is produced in which cycles are repeated again and again. The word 'alternation' never means derivative with respect to time i.e. $\frac{d}{dt}$. But Newton did not write the word 'alternation' as derivative with respect to time i.e. $\frac{d}{dt}$. Did Newton write this notation at some other stage? For this related works of the legend and other scientists are to be studied. When Newton defined the second law of motion in the Principia, then even mathematical equation F =ma or F =Kma was not quoted at all. When these equations were quoted ? It has to be assessed whether Newton or other scientists quoted these equations involving time derivatives. The answer to this question can be found by studying relevant literature.

To justify the second law of motion, following short explanation was given by Newton after stating second law of motion in *the Principia.*

"If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively. And this motion (being always directed the fame way with the generating force) if the body moved before, is added to or subtracted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both".

When Newton stated second law of motion in the Principia [1] at page 19 , then derivative $\frac{d}{dt}$ was not used or mentioned , and even F=Kma or F=ma was not quoted. It is pertinent to mention that Newton was also inventor of calculus. Thus legend knew about functions of derivatives

From definition of second law of motion

Alternation in motion α Motive Force Impressed

In no way Sir Isaac Newton wrote (while quoting second law of motion in the Principia)

alternation in motion = rate of change in momentum ($\frac{dp}{dt}$) (8.6)

Anyhow let us move with the text quoted in the numerous publications. Thus,

$\frac{dp}{dt}$ α Motive Force Impressed

To remove the sign of proportionality, a coefficient of proportionality is introduced and its value is determined experimentally. If its value remains the same in all cases then coefficient of proportionality is called 'constant of proportionality.' In the existing literature there are no evidences that ever an attempt was made to measure the value of K experimentally.

$\frac{dp}{dt}$ = k Motive Force Impressed

$\frac{dp}{dt}$ = k F

or F = $\frac{1}{k}\frac{dp}{dt}$

or kF = ma

or F = Kma (5)

It is already mentioned that when Newton defined second law of motion F=Kma or F =ma , were not written hence question of determination of K did not exist. The value of K may have been assessed at later stage. English scientist Henry Cavendish experimentally measured the value of universal gravitational constant G in 1798, however law of gravitation was stated in 1687,

In the existing literature the value of K is regarded as unity in the following way. It is assumed that if a body has unit mass (m=1) and unit acceleration (a=1), such that force possessed by it is also one unit (F=1) then value of K is unity. Now eq.( ) becomes

m =1 , a=1 , F=1

1=K$\times$1$\times$1

or K =1 (6)

In motion of bodies resistive forces (atmospheric, frictional and gravitational etc) , play significant roles, this fact is not taken in account. If the magnitude of resistive forces is large, then more force is required to produces acceleration of $1 m/s^2$ in body of mass 1kg.

Did Newton or any other scientists measured or assumed value of K as unity. What is the magnitude or degree of resistive forces when value of K is measured? The answer to this question can only be found by studying the related literature. Newton's second law of motion is applicable for all bodies of different masses in ideal and practical media. If the resistive forces are infinitesimally negligible then with smaller force body can move larger distance and the resistive forces in the medium are maximum, then body will travel the least distance. F =Kma is applicable to various masses (not for m=1, used in determination in value of K), some of them are

m_e (mass of electron) = 9.1×10^{-27}kg , mass of man = 78kg

mass of elephant = 300kg, mass of the sun = 1.9891×10^{30} kg

and mass of Jupiter = 1.89813×10^{27} kg

Similarly Newton's second law of motion is applicable to various values of accelerations , however value of K is only determined for unit acceleration and body of unit mass.

The method of assumption are not scientifically valid method for determination of coefficients of proportionality in science. Now if this method is applied to all equations in physics, then all values of coefficients of proportionality in Physics will be regarded as unity. We will discuss its implications in measurement of universal gravitational constant G in this case. If the same method is used in determination of value of G , then

$$F = G\frac{mM}{r^2}$$

$$1 = G\ 1 \times 1/1 \quad \text{or } G=1 \qquad (7)$$

which is not justified as all astronomical data (including mass of earth, distance between earth and sun) will change drastically.

Similar is the situation of co-coefficient of viscosity and other coefficients .

$$F = \eta\ A\frac{dv}{dx}$$

$$1 = \eta \times 1 \times \frac{1}{1}$$

$$\text{or} \quad \eta = 1 \qquad (7)$$

Thus the way the value of K is regarded as unity in F=Kma, leads to inconsistent results if applied to other cases.

4.0 Prediction of undefined inertial mass from equation F =ma or $m = \frac{F}{a}$

Inertial Mass : When a body is in translation under effect of external force other than gravity , the mass of body measured is called its inertial mass.

Let force F produces acceleration a in the body of mass m, then mathematically inertial mass is given by

$$m = \frac{F}{a} \qquad (9)$$

Inertial mass when motion is unaccelerated (a=0, v=u, F=0)

Newton's Second Law of motion (F=ma) implies that when no external force (F=0) acts on the system, then body moves with uniform velocity (u=v , a =0). Thus,

F=0 , a =0

Thus Newton's first law of motion follows from the second law of motion. Under similar conditions Newton's third law of motion follows from the first law. So Newton's second law of motion is real law of motion. Now the equation for inertial mass i.e. eq.( ) becomes

$$m = \frac{0}{0}$$

It is reiterated that under this condition (F =0, a=0) , the second law of motion reduces to first law of motion. But under this condition (F =0, a=0) inertial mass becomes UNDEFINED, hence all other interpretations are meaningless. This aspect is not discussed.

Dimensions of LHS = M

Dimensions of RHS = undefined

Units of LHS = kg (SI system)

Units of RHS = undefined (in every system)

Although division by zero is not allowed, yet it naturally follows from mathematical equation based upon Newton's second law of motion. In addition in this case numerator also becomes zero under feasible condition i.e. when F=0, a =0 (u=v, un accelerated motion). So it is only and only limitation of Newton's Second Law of Motion, but not discussed in the existing literature.

It can be explained (mass is well defined , m>0) if acceleration is non-zero thus force is also non-zero. Thus external force is required for movement of body and non-zero acceleration is produced. This hints validity of Aristotelian concept of motion [2] and force but acceleration was not defined in time of Aristotle.

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References

1. Newton , Isaac *Mathematical Principles of Natural Philosophy* pp.19-20 , London, 1727 , translated by Andrew Motte from the *Latin*.
2. Sharma , Ajay *Newton and Archimedes* , *Now and Then* , to be published.